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M&S Research Hub institute
Carlo Mierendorff Str. 43, 34132 Kassel, Germany
Telephone +49 (0)56149941680,
Email: info@ms-researchhub.com,
Web: www.ms-researchhub.com

On the Welfare Trends: A view from the Sen's Social Welfare Function

Riveros-Gavilanes, John Michael[1]
Fayq Al Akayleh[2]
Oduniyi, Oluwaseun Samuel[3]
Azar Huseynli[4]
Sherif M. Hassan[5]

Abstract:

In this article, we use the theoretical construction of Sen's Social Welfare Function to estimate the trends of welfare for different world regions. The empirical analysis is also complemented by investigating the long-run relationships and the role of income inequality on welfare dynamics. The results suggest heterogeneous relationships across regions between income, inequality, and welfare. Implying that welfare across regions is not similarly influenced by unequal distribution of income, while some regions are positively correlated with improvements in the income distribution leading to better welfare measures. Over the short run, economic growth is the dominant factor in steering welfare.

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1 M&S Research Hub (Kassel, Germany). Corporación Centro de Interés Público y Justicia CIPJUS (Bogotá, Colombia), Estudios y Evaluación de la Gestión Pública Colombia (EEGPC) https://orcid.org/0000-0003-4939-0268

2 Associate Professor of Economics, Head of Research and Investment Analysis Department, Falcom Holding, Adjunct Professor of Economics at Arab Open University,Riyadh, Saudi Arabia.

3 Texas Tech University, Department of Agricultural and Applied Economics, 2500 Broadway, Lubbock, TX 79409.

4 Lecturer,UNEC Business School, Azerbaijan State Economic University(Baku,Azerbaijan) https://orcid.org/0000-0003-2109-0810

5 M&S Research Hub, 34132 Kassel, Germany; Economic Research Forum (ERF), Cairo 12311, Egypt. British University Egypt (BUE), Suez Canal University (Egypt)

1 Introduction

Vast empirical studies show that the world is facing increasing levels of income inequality. Important questions regarding the welfare impact of inequality, and whether positive income growth rates can mitigate the negative effects of inequality remain unaddressed. In this paper, we use the theoretical construction of the social welfare function proposed by Sen (1974) to determine the welfare trends across world regions and we link this behavior to the evolution of inequality within an empirical approximation to estimate the welfare elasticities of income and inequality for each region.

Using panel data of the 7 world regions, the regression equation is based on the theoretical formulation of Sen (1974) to reveal the relationships between the variables in the equation. Some papers use the traditional estimation of the Sen welfare index but only one paper uses the same underlying econometric procedures (Riveros-Gavilanes, 2021). The econometric model is based on the generalization process of the Sen (1976) SWF provided by the studies Mukhopadhaya (2001) and used in Riveros-Gavilanes (2021) which relies on the panel fixed-effects specification.

We complement the econometric analysis by studying the cointegration between the average income of the economies, the levels of income inequality, and the welfare around the different regions of the world, based on the Sen's Social Welfare Function -SWF-. The results indicate that cointegration patterns and the impact of inequality on welfare are heterogeneous across the regions.

2 Literature Review

Welfare (well-being) is considered as one of the main aspects of economic development and it is argued that economic development will be undermined if the policymakers fail to secure the required level of welfare following the level of economic growth (Awan, 2015).

This research aims to analyze empirically the global and regional trends of welfare by using the Social Welfare Function -SWF- proposed by Sen (1956), yet it is essential first to identify the proper measures of (social) welfare. These basics are associated with three fundamental theorems. The first is the Pareto efficiency, which states that under certain assumptions, competitive markets produce efficient outcomes, i.e., Pareto efficiency (Jean and Gareth, 2013), capturing Adam Smith's invisible hand (Andreu et al, 1995). The second pertains to the neoliberalists' argument about the linkage between equilibrium and welfare, which states that any Pareto efficient outcome can be conceptualized as a competitive market equilibrium (Stiglitz, 2018). The third fundamental theorem is Arrow's Impossibility Theorem of Social Choice (Arrow, 1963; Feldman, 2008), which states that (U) the social welfare function is defined for all profiles; (P) if the profile is such that everyone prefers alternative a to alternative b, the social ordering sets an above b (Pareto efficiency); (I) the choice between two alternatives depends only on the individual preferences (independence of irrelevant alternatives); and (ND) the social ordering does not always concise with the preferences of any single individual (non-dictatorship). Arrow then showed that there is no social welfare function satisfying all of these conditions (Impossibility).

The basic notion on SWF is that it is analogous to the individual consumer's Indifference Curve Map (ICs), except that the ICs represent individual consumer's preferences whereas SWF represents a society's preferences (collective social choice). Inputs of SWF can include any variables that are believed to affect the economic welfare of a society (Sen, 2018). The first published work on SWF is the Bergson-Samuelson SWF (Igersheim, 2019). Bergson (1938) who have introduced the first notion of the Bergson-Samuelson SWF, which has been further developed by Samuelson (1947). Bergson SWF was established to rank all variables on which the individual welfare depends; those variables include the quantities of different commodities produced and consumed and of production resources used in the production of different commodities.

Arrow's impossibility theorem has inspired vast literature, including the Noble Laureate Amartya Kumar Sen. A widely used SWF was that of the Lorenz approach to measuring the concentration of wealth and, hence to demonstrate the income inequality (Lorenz, 1905). The work of Lorenz (1905), however, was criticized based on two shortcomings; first, the Lorenz approach does not give complete orderings of social states and second, the used SWF in the Lorenz approach, welfare can be comparable only when mean incomes are equal.

Many researchers attempted to overcome problems associated with Lorenz's approach by indicating a way of ranking alternative social states. Atkinson's theorem has been extended to allow social welfare evaluations of income distribution with unequal mean incomes (Shorrocks, 1983; Kakwani, 1984; Kakwani, 1988). The generalized Lorenz curves (by Kakwani and Shorrocks) are constructed by scaling up the Lorenz curve by the mean of the distribution. Thus, the height of the generalized Lorenz curve reflects the level of income, while the convexity of the generalized Lorenz curve reflects the degree of income inequality.

We summarize in Table 1 findings from the literature about the empirical estimations of the welfare trends, income, and inequality levels based on Sen's perspective.

Table 1. Empirical Studies and recent development of welfare trends

Author	Study	Sample	Description
Sen (1976)	Real National Income	Indian's Intra-State data for 1961 and 1962	Using the SWF developed by himself, Sen ranks the states of India to determine the trends of welfare
Berrebi & Silver (1987)	Regional Differences and the Components of Growth and Inequality	United States, cross-sectional data in a time series framework for 1960, 1970, 1980.	The trends and relative changes over time for the United States are estimated using the Sen's SWF traditional approach, and they discuss the relative change in welfare over time.
Gasparini & Sosa Escudero (2001)	Assessing Aggregate Welfare: Growth and Inequality in Argentina	Time Series sample for Argentina between 1980 and 1998.	The study implemented different welfare indices for Argentina and their growth in time.
Mukhopadhay (2003a)	A Generalized Social Welfare Function and Its	Time Series sample, for welfare trend estimation for	This is the pioneer study where the generalization of the Sen's Social Welfare

	Disaggregation by Components of Income	Australia from 1984 to 1994	Function was first established and analysis of national income components.
Mukhopadhaya, (2003b)	The Ordinal and Cardinal Judgment of Social Welfare Change in Singapore,	Monthly Time Series sample for Singapur between 1982 to 1999.	Analyzing the establishment of welfare ranked with Lorenz dominance technique and Sen's Social Welfare Function
Baluch and Razi (2007)	Social Welfare Measurement in Pakistan: An Ordinal and Cardinal Approach	Individual Time Series sample for Pakistan between 1980 to 2002.	Analysis of income inequality by using the Gini coefficient and the welfare trends using the original and generalized Sen's SWF.
Bishop et al (2009)	Practitioner's Corner: An Asymptotically Distribution-Free Test for Sen's Welfare Index	Cross-sectional microdata of households for the selected the year 1980 in the U.S.	The article estimates Sen's SWF results and compares them with an asymptotical approach from inequality in the sample.
Mukhopadhaya (2014)	Income Inequality in Singapore	Individual Time Series Sample of Singapore from 1984 to 2011.	The article uses the generalization of Sen's SWF to analyze the ranking of welfare for Singapore in the years of study.
Riveros-Gavilanes (2021)	Estimating the Social Welfare Function of Amartya Sen for Latin America	Panel Data consisting of 15 Latin-American countries between 1995 and 2018	The study presents the welfare trends for each country evaluated at different values of the Pareto's efficiency parameter and introduces the first regression approach of Sen's SWF.

Source: Own Elaboration.

It is noted that most of the studies rely on the traditional Sen's Welfare Index. These estimations are unambiguously presented under the name Sen's Index and typically rely on cross-sectional or time-series data to calculate the welfare based on the perspective of Amartya Sen. There is a gap in the literature on investigating the welfare impact of inequality both on the spatial side, by applying on a panel of world regions, and also on the methodological side by applying the generalization process of the Sen (1976) SWF index. Only one study by Riveros-Gavilanes, (2021) has resembled this study by estimating the SWF for Latin America. The results indicated that there are long-run relationships between income, inequality, and welfare (proxied by Human Development Index) to estimate the Pareto-efficient parameter. Based on the empirical findings, it was postulated that Latin America has no strict Pareto Optimal principle in the welfare function to define the trade-off between efficiency and equity.

3 Theoretical framework

The Sen's Welfare Index has the ground foundation in the Lorenz curve and the generalized version of this curve. Sen (1954) develops the axiomatic introduction of the social welfare function which further in time Mukhopadhaya generalizes and will be used in this study to estimate the welfare trends under this approach. Hence, this part presents a theoretical description of the conceptual elements required to assess Sen's social welfare function approach for the estimation.

A Lorenz curve is a function L: [0,1]→[0,1] that satisfies (i) non-decreasing on [0,1], (ii) convex on [0,1], (iii) L(0) = L(0+) = 0, and (iv) L(-1) = L(1) = 1. The Lorenz curve was derived as shown by the following form:

$$L_i(p) = (\mu_i - \alpha_i)^{-1} \int_0^p [X_i(u) - \alpha_i] du \qquad (1)$$

where, $L_i(p)$ is the Lorenz function and, hence Lorenz curve, p ∈ [0,1], μ stands for the mean of function variable (e.g., income) and is assumed to be finite and positive,

α_i = sup {x : Fi(x) = 0} and βi = inf{x : Fi(x) = }, i = 1,2. αi and βi denote the minimum and maximum attainable incomes and,

Xi(p) is the order function (quantile or fractile function). The Xi(p) are left-continuous and non-decreasing, with Xi(0) = αi and Xi(1) = βi. If $X_1(p) \geq X_2(p)$ for all p∈ [0,1], then the distribution F1 order dominates the distribution F2, denoted as F1 ≥0 F2.

The generalized Lorenz curve is then defined by Shorrocks and Kakwani as the ordinary Lorenz curve multiplies by the mean of Fi. That is,

$$G_i(p) = \int_0^p X_i(u) du \qquad (2)$$

$$G_i(p) = (\mu_i - \alpha_i) L_i(p) + \alpha_i P$$

The generalized Lorenz curve is continuous and convex on [0,1]. It also satisfies G(0) = 0 and Gi(1) = μ_i. It should be noted that $G_i(p)$ will not take a negative value unless there is at least one individual with negative income. If $G_1(p) \geq G_2(p)$ for all p∈ [0,1], then the distribution F_1 generalized Lorenz dominates F_2, denoted $F_1 \geq_G F_2$.

The social welfare implications of the above-discussed distributions is that we are comparing income distributions for populations, say, of N individuals. If $F_1 \geq F_2$, then $F_1 \geq(1)$ F_2, i.e., if income distribution F_1 is greater than income distribution F_2, this means that social welfare is higher under distribution F_1 (i.e., $W(x_1) \geq W(x_2)$) for all increasing, symmetric social welfare functions. And, if $F_1 \geq F_2$, then $F_1 \geq_{(S)} F_2$, i.e., if income distribution F_1 is greater than income distribution F_2, this means that social welfare is higher under distribution F_1 for all increasing, S-concave social welfare functions. The literature has proven this outcome (Thistle, 1989).

As far as income inequality is concerned, the ordinary Gini coefficient is commonly used, which can be computed as twice the area between Lorenz curve and the 45° equality line, i.e.,

$$\gamma\mathrm{i} = 2\int_0^1 [p - Li(p)]dp \qquad (3)$$

for the case of generalized Lorenz curve, the quality line is the line from the origin point to the point $G_i(1) = \mu_i$. The Gini coefficient can be computed from the generalized Lorenz curve by multiplying and dividing the above equation by μ_i, that is,

$$\gamma i = \left(\frac{2}{\mu i}\right)\int_0^1 [\mu i p - Gi(p)]dp. \qquad (4)$$

The Gini coefficient is one-half the Gini mean difference, divided by the mean. It follows that, twice the area between the 45° equality line and the generalized Lorenz curve (i.e., $\mu_i P$) is one-half the Gini mean difference for the distribution. This area then gives an absolute measure of income dispersions.

The extended Gini coefficient is related to the relative poverty line, which was introduced by (Kakwani, 1980 and Sen, 1976). The extended Gini coefficient for the distribution F_i can be defined as follows:

$$\Gamma i(r) = \int_0^\beta [1 - Fi(x)]\, r dx \qquad (5)$$

where $r \geq 0$, is an inequality aversion index.

The extended Gini index can be used as a proxy for an absolute inequality index. For r = 0, $\Gamma_i(0) = X_i(1)$, which is the maximum attainable income, and for r = 1, $\Gamma_i(1) = \mu_i$, which is equal to the mean minus one-half of the Gini mean difference, for r = 2, $\Gamma_i(2) = \mu_i(1-\gamma_i)$, and for $r = \infty$, $\Gamma_i(\infty) = X_i(0)$, which is the lowest attainable income. To sum up, the extended Gini coefficient is closely related to stochastic dominance.

However, Shorrocks's (1983) work of the generalized Lorenz criterion was criticized for its extreme paretianity, i.e., when considering that the welfare of society increases as the incomes of the richest income group increase regardless of others' incomes (Tam and Zhang, 1996). Alternatively, Tam and Zhang (1996) developed a set of SWFs presented empirically by dominance relationship, allowing for trade-offs between efficiency and equity and their functions can be shown as:

$$W^{\beta}(y) \geq W^{\beta}(x) \text{if } \frac{\sum_{i=1}^{h} y_i}{\sum_{i=1}^{h} x_i} \geq \left(\frac{\mu_y}{\mu_x}\right) 1-\beta,, h = 1, 2, \dots, N \text{ and } 0 < \beta < 1. \tag{6}$$

where, μ is the mean income of, say two countries (x and y) and β is a proxy for the trade-offs between efficiency and equity. When $\beta = 1$, β-generalized Lorenz dominance is simply the generalized Lorenz dominance and, when $\beta = 0$, β-generalized Lorenz dominance is clearly the ordinary Lorenz dominance. The ordinary Lorenz dominance is equity-biased (Rawlsian) and the generalized Lorenz dominance is efficiency-biased (extreme Paretian). SWFs and ranking based on these criteria of dominance are incomplete (Mukhopadhaya, 2001).

Using the previous theoretical elements, and assuming a negative relationship between society marginal utility and income rank, Sen (1974) introduced a new axiomatic form of the SWF,

$$W = \mu(1 - G) \tag{7}$$

where, μ is the society's mean income and G is the Gini coefficient of the income distribution. Since Gini coefficient is defined as twice the area between the 45° equality line and the ordinary Lorenz curve, it follows that (1 – G) is twice the area below the ordinary Lorenz curve. Sen's index tries to consider both efficiency and equity criteria (i.e., Pareto optimality and Rawlsian idea). By letting L(p) be the ordinary Lorenz curve, the generalized Lorenz curve is then,

$$GL(p) = \mu L(p). \tag{8}$$

It follows that, Sen's SWF is twice the area below the generalized Lorenz curve, i.e.,

$$= 2\int_0^1 GL(p)dp = \mu(1 - G) \tag{9}$$

Sen's SWF stated above was criticized on its rigidity with respect to efficiency (Pareto optimality) and equity (Rawlsian rule). The rigidity level can be examined by the marginal rate of substitution between efficiency and inequity, which (for Sen's SWF) is

$$\frac{dG}{d\mu} = \frac{1 - G}{\mu} \tag{10}$$

Mukhopadhaya (2001) argued that Sen's SWF is an extreme Paretian SWF where, as depicted by the above-derived equation, it is highly sensitive to mean income and less sensitive to inequality and, hence biased in favor of developed countries which have higher per capita income and relatively low-income inequality than developing countries. After allowing for the coefficient of marginal rate of substitution between efficiency and equality to be changing, Mukhopadhaya (2001) suggested the following SWF,

$$W = \mu^{\beta}(1 - G) \tag{11}$$

where $0 \leq \beta \leq 1$.

This suggested Sen-type SWF allows the decision-makers to have the choice between efficiency and equity captured in the Pareto trade-off parameter β, yet need to be theoretically proved and empirically examined. Riveros-Gavilanes (2021) used the original SWF of Sen and the generalized Sen's SWF of Mukhopadhaya to establish a set of empirical approximations and estimates the welfare trends in Latin American countries.

4 Methodology

4.1 Traditional Estimation of the Welfare Trends

Based on the original social welfare function of Sen (1974), we derive a panel setting of the welfare trends as follow

$$W_{it} = y_{it}(1 - G_{it}) \tag{12}$$

Where the i-th individual is composed by the countries from $i = 1,2, \dots, N$ in the sample, and t periods. The sample of the study consists of a panel data set at a country level for the world (over 140 countries) for the years from 1995 to 2018 in an annual periodicity, in total around 1452 observations are available for the estimation.

According to Sen (1976), Arrow (1999), Mukhopadhaya (2001), and Riveros-Gavilanes (2021) this traditional estimation setting is acalculous of the average income level of the economy y (measured for our case by the real GDP per capita) and the complement of the Gini coefficient $(1 - G_{it})$, where G represents the levels of income inequality measured by Gini. The logic behind Sen's welfare index is to weigh the income levels by the respective size of inequality, and this is extended to represent countries over time. The result is a measure of welfare in the units of the average income of the economies (proxied by real GDP per capita in USD).

4.2 Econometric Estimation of the Sen's Index

The econometric extension is based on the theoretical generalization process of the Sen (1976) SWF represented in equations (7) and (12) that was first introduced by Mukhopadhaya (2001) and finally expressed in equation (11). Similar to Riveros-Gavilanes (2021), we add fixed effects to the principal equation as follow:

$$\ln W_{it} = \beta_0 + \beta_1 \ln y_{it} + \beta_2 \ln(1 - G_{it}) + \mu_i + u_{it} \tag{13}$$

Equation (13) is a linearization of the generalized Sen's SWF from equation (11) using natural logarithms[6], with the elasticity components β_1, and β_2. The term μ_i are the country fixed effects and u_{it} is residual in the panel data structure. The dependent variable W_{it} is a welfare measure, which for our study -and following the same suggestion of Riveros-Gavilanes (2021)- is proxied by the Human Development Index -HDI-. Equation (13) represents the structural approach of the relationships described by Sen's axiomatic SWF which constitutes the foundation of the model. The set of explanatory variables are the mean income of the economies (in real terms measured by the GDP per capita) as y, and the complement of the Gini coefficient $(1 - G_{it})$ where G_{it} represents the Gini Coefficient as the measure of income inequality. The research hypothesis, following the original representation of Sen's function, that both variables; the mean income of the economies and the inequality levels of income are correlated with the levels of welfare, thus parameters β_1 and β_2 are expected to be statistically significant, and positive.

The specification above represents a long-run approximation since it only contains contemporaneous values of the variables, the original econometric research implied the use of cointegration by Riveros-Gavilanes (2021) for Latin America only. In this research we extend the analysis to the world regions, implying that N>T. This raises a difficulty to detect long-run relationships, but for our research, we will segment the regressions and the tests across the regions the cointegration hypothesis, We follow the World bank classification of world's region as shown in Table 2:

Table 2. World Bank Classification of Regions.

Regions of the World
East Asia and Pacific
Europe and central Asia
Latin America and the Caribbean
Middle East and North Africa
North America
South Asia
Sub-Saharan Africa

Source: World Bank (2021)

For the econometrical procedure, the choice of a welfare proxy is not straightforward, and the literature provides multiple resources where each differs regarding the definition of welfare. In this study we will proceed as suggested by Riveros-Gavilanes (2021) using the Human Development Index, as a multidimensional measure for the welfare that considers income, education and health.[7] The second reason for using HDI is the availability of the data for our sample of world regions. The sources of information are the World Bank (2021a), the International Monetary Fund (2021), the United Nations Development Program -UNDP- (2021), and the Penn World updated in 2021 (Feenstra, Inklaar and Timmer, 2015).

6 The mathematical model is in fact specified as $W_{it} = y^{\beta_1}(1 - G_{it})^{\beta_2} e^{\beta_0 + \mu_i + u_{it}}$

7 The inequality adjusted version of the HDI, was considered during the empirical analysis but given the lack of data for the years of the study it was discarded. This is because the United Nations Development Program only provides world-wide estimations of this adjusted version starting the year 2019. We argue that important inferences can be derived by using the HDI for the world-wide analysis with large scale data to detect possible relationship with the inequality and the welfare rather than reducing drastically the sample to a specific year. Thereby, the HDI will provide the analysis of this dynamics over time.

We test for presence of unit root by using first- and second-generation test (see Appendix A), then we test for cointegration. When cointegration exist between the variables for a specific world region, we run an error-Correction model to detect short and long run dynamics using the following setting

$$\Delta \ln W_{it} = \pi_0 + \pi_1 \Delta \ln y_{it} + \pi_2 \Delta \ln(1 - G_{it}) + \delta ECT_{i,t-1} + v_{it} \quad (14)$$

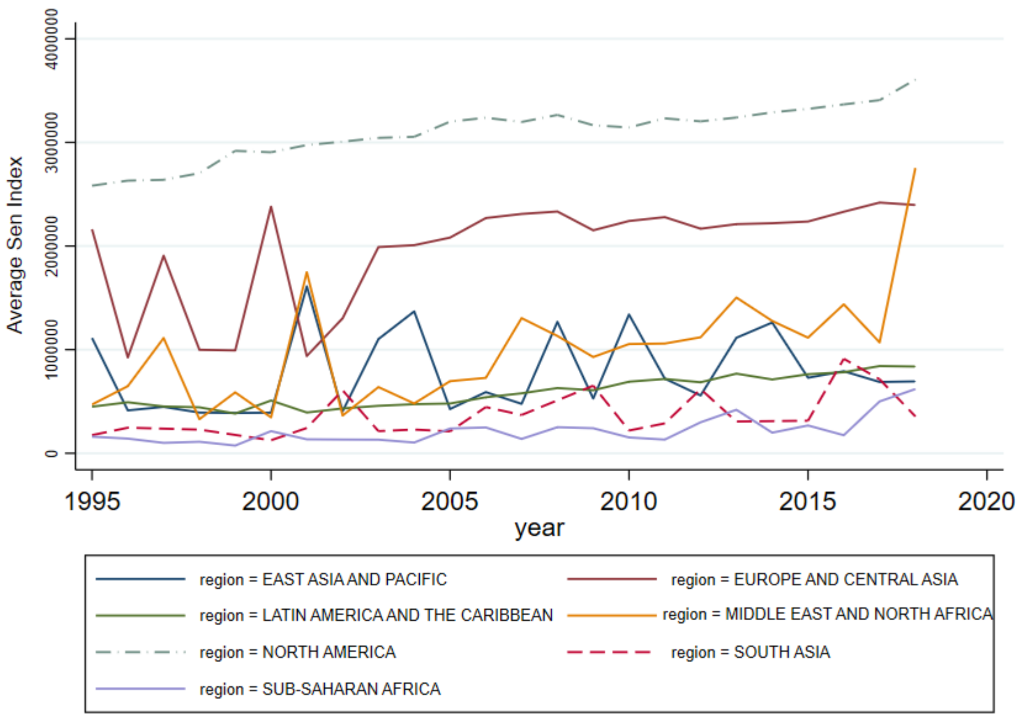
Where δ is the coefficient of the error-correction term ECT which contains the lagged residuals of the long-run equation u_{it}. A stable process towards the equilibrium is reflected when the parameter δ is statistically significant, negative, and between zero and one. This will work to support the idea of a long-run process between the average income level of the economies and the complement of the Gini coefficient with welfare. The short-run model can be understood as a growth equation since it contains the growth of Welfare, the economic growth, and the growth in the complement of the Gini coefficient with the error-correction mechanism towards the long-run equilibria.

5 Results

5.1 Traditional Estimation of the Sen's Welfare for the regions of the World

The overall trends of welfare across world regions measured by the traditional Sen (1974) SWF are presented in the next graph. It is noted that in general, welfare exhibits an increasing trend.

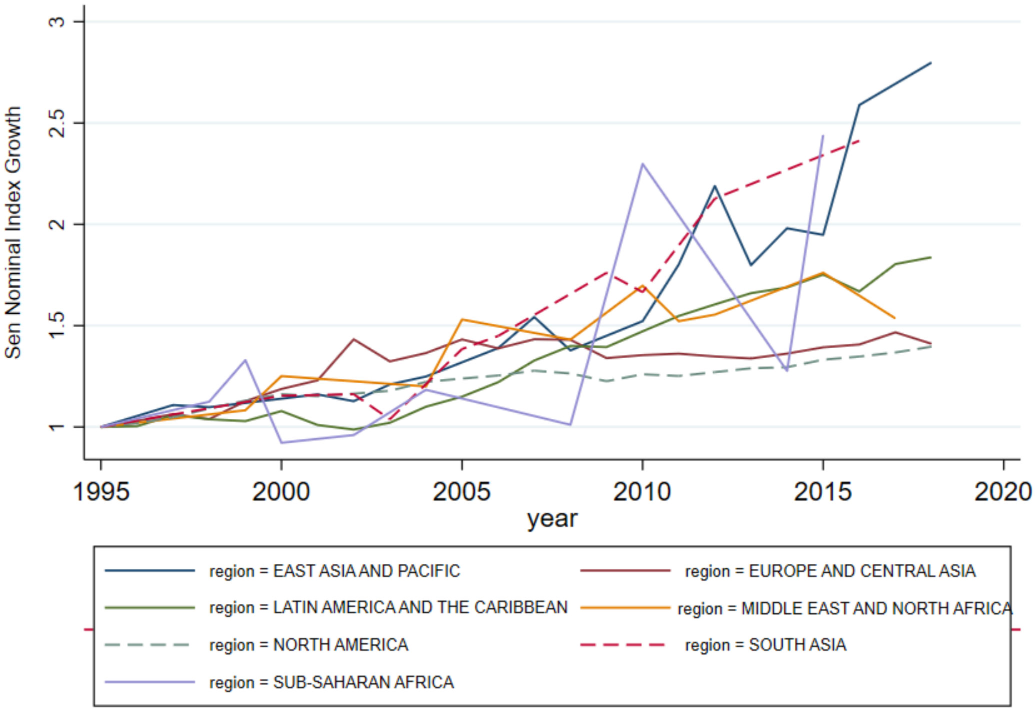
Figure 1. Sen's Welfare Trends Estimates - Average by region



Source: Own Elaboration

To better inspect the evolution of the welfare, we compare under a normalized starting point, the nominal growth of the result of the Sen's SWF in Figure 2. Wherein the initial value of the index is used as a reference to inspect the individual-region evolution.

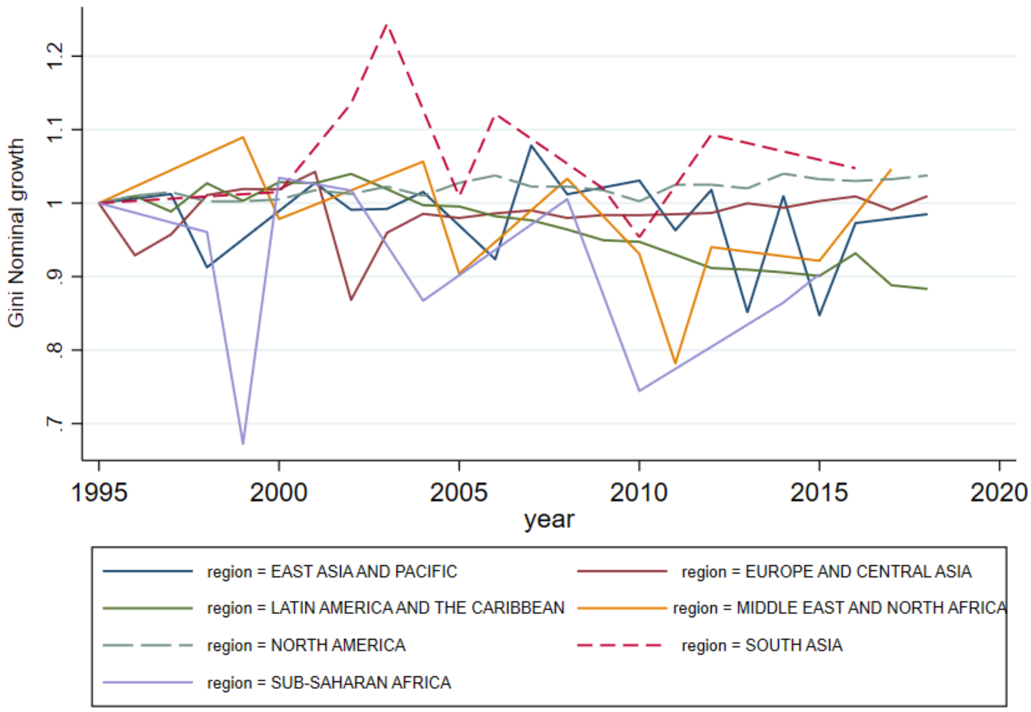
Figure 2. Sen's Welfare Estimates Nominal Evolution - Average by region



Source: Own Elaboration

The general trends are positive in what relates the average nominal evolution of the welfare by region. The region which exhibits the larger growth of welfare from its starting point is East Asia and Pacific, this witnessed in figure 2 by comparing the year 2018. The region which presented the lowest growth in nominal terms is North America, and can be characterized over time as a the region with the lowest nominal acceleration in terms of the Sen's welfare index, besides that it is also the region with the higher levels of welfare. The region of Middle East and North Africa presents from 2015 onwards a reduction in the growth of its welfare trends. The area with the largest erratical nominal growth of welfare is Sub-Saharan Africa, where there are abrupt changes in their welfare trends in multiple periods of time involving the years 1998, 2000, 2010 and 2014.

Figure 3. Gini Nominal Evolution - Average by region

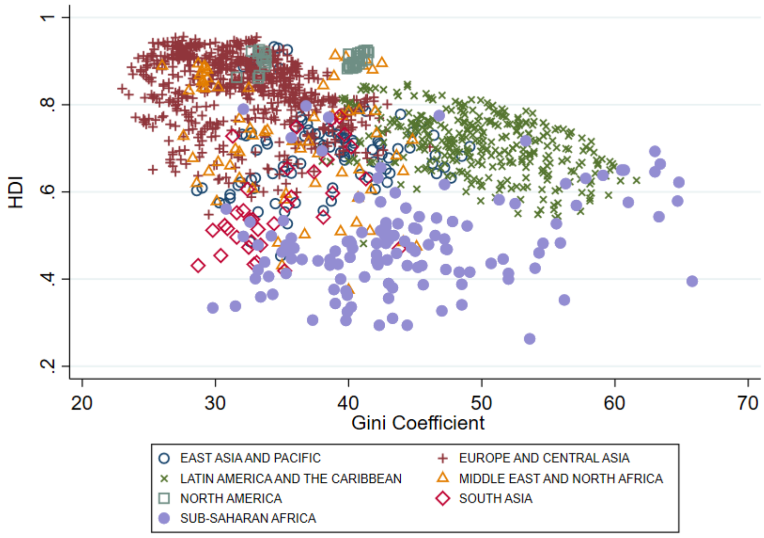


Source: Own Elaboration

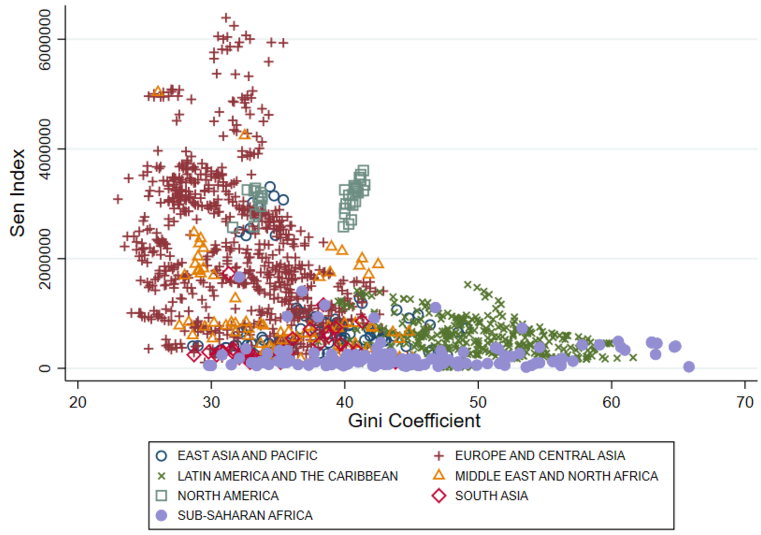
The interpretation of the graph is that if the nominal growth of inequality (measured by the Gini and using regional averages of the country data) equals 1, the initial value of the income inequality remains unchanged. if the nominal growth is is less than 1, this begets an improvement in the income distribution and vice versa with growth values larger than 1. As shown, income inequality has overall increased across South Asia, North America, the Middle East and North Africa, Europe, and Central Asia. This approach highlights that advances in welfare measures have been first triggered by changes in the income levels rather than the improvements in the income distribution.

Figure 4. Scatterplots HDI and Sen's SWF vs Gini

(Panel A)



(Panel B)



Source: Own Elaboration

The largest welfare inequality can be found in Europe and Central Asia. On the other hand, regions like Sub-Saharan Africa exhibit a large variance of income inequality relative to low levels of welfare inequality. It is noted how lower levels of welfare by the Sen's Index tend to be correlated with higher income inequality values, particularly in regions of Latin American and the Caribbean with Sub-Saharan Africa. These facts are also observed in the traditional estimation of the Sen Welfare Function presented in the next table.

Table 3. Traditional Sen SWF Estimation Descriptive Statistics

Region	N	Min	Max	Mean	P50	Sd	Variance
North America	36	2571597	3606886	3106122	3161593	268885.9	7.23e+10
Europe and central Asia	721	108079.6	6391400	2152424	1855164	1329596	1.77e+12
Middle East and North Africa	77	99987.25	5034672	1040331	754470.4	890511.1	7.93e+11
East Asia and Pacific	98	164770.6	3311371	814228.4	587547.7	725579.7	5.26e+11
Latin America and the Caribbean	334	23499.57	1530900	589979	557457.4	316479.8	1.00e+11

South Asia	35	107994.1	1730647	398081.7	296041.8	328032.1	1.08e+11
Sub-Saharan Africa	151	23174.07	1664385	210679.1	123315.7	248381.6	6.17e+10
Total	1452	23174.07	6391400	1423152	913474	1301175	1.69e+12

Source: Own Elaboration

Table 4. Gini Coefficient Descriptive Statistics

Region	N	min	max	mean	p50	sd	variance
Latin America and the Caribbean	334	38	61.6	49.82814	49.8	5.021657	25.21704
Sub-Saharan Africa	151	29.8	65.8	44.17682	42.8	8.101896	43.64073
North America	36	31.6	41.5	38.21667	40.3	3.576551	12.79171
East Asia and Pacific	98	28.6	49.1	37.96837	37.95	4.81104	23.1461
Middle East and North Africa	77	26	45.1	35.34156	35	5.115131	26.16457
South Asia	35	28.7	43.8	34.9	33.4	3.787829	14.34765
Europe and central Asia	721	23	46.1	32.0785	32	4.487402	20.13677
Total	1452	23	65.8	38.21033	36.1	8.96043	80.28931

Source: Own Elaboration

Table 5. Real GDP per capita in USD (2011) Descriptive Statistics

region	N	min	max	mean	p50	sd	variance
North America	48	36212.39	61550.96	48753.27	48366.43	6253.64	3.91e+07
Europe and central Asia	1121	1472.394	92763.42	27612.17	23742.18	18665	3.48e+08
Middle East and North Africa	464	1273.541	141850.6	27611.63	12198.59	29639.98	8.79e+08
East Asia and Pacific	381	935.7791	85276.4	22979.53	12113.59	22125.82	4.90e+08
Latin America and the Caribbean	792	355.0846	36095.09	12225.27	11019.58	7469.889	5.58e+07
South Asia	168	1620.423	28245.55	6671.068	4293.854	5802.273	3.37e+07
Sub-Saharan Africa	1077	462.9643	46255.66	4576.144	2367.933	5992.136	3.59e+07
Total	4051	355.0846	141850.6	17425.84	10629.71	19342.3	3.74e+08

Source: Own Elaboration

5.2 Econometric Estimations of the Sen's Welfare Index

The variables HDI, GDP per capita, and the natural logarithm of Gini's coefficient complement exhibit unit-root in levels but become stationary at the first difference (Appendix A). The SWF regression estimates using the worldwide data against HDI are presented in the next table using three additional controls for the regressions (Unemployment Rate, Exchange Rate National Currency/USD, and Total Investment as % of GDP). The sample utilizes data from 142 countries. The results show that both the GDP per capita and the complement of the Gini coefficient are positively correlated with the levels of welfare measured by the HDI. These results are robust across the Driscoll-Kraay robust standard errors, fixed effects, and random effects specifications.

Table 6. Worldwide regressions over the Human Development Index

	(1)	(2)	(3)
VARIABLES	World FE (DK)	World - FE	World - RE
ln_GDP_cap	0.219***	0.219***	0.210***

	(.0083023)	(.0035004)	(0.00336)
ln_Gini_complement	0.0728**	0.0728**	0.076**
	(.0105093)	(.0125879)	(0.0126)
Constant	-2.704***	-2.704***	-2.616***
	(.098)	(.0524957)	(0.0670)
Observations	1190	1190	1190
R-squared Overall	0.8277	0. 8277	0. 8277
R-squared Within	0.8284	0.8283	0.8283
Number of groups	89	89	89
Country Fixed Effects	Yes	Yes	Yes
Controls Included	Yes	Yes	Yes

Note: Dependent Variable (ln_HDI). Controls included used the Unemployment Rate %, the Exchange Rate at national currency in USD, and the total investment as a percentage of the GDP Standard errors are in parentheses

*** p<0.01, ** p<0.05, * p<0.1 Source: Own Elaboration

The natural log of GDP per capita is statistically significant at a 1% level of significance, where a one-unit increase, raises HDI by 0.22% in average ceteris paribus. On the other hand, the logarithm of the Gini complement is statistically significant at a 5% level of significance, wherein an increase of 1% in the complement, fosters HDI by 0.073%. The linear adjustments of the models vary from 82.84% within countries to an overall of 82.77% as a whole.

We now proceed to the regional results to investigate possible heterogeneous correlations. Since cross-sectional dependence is a topic that can be troublesome in this setup, we used Driscoll-Kraay's robust standard errors to provide a more accurate statistical inference during the regressions, to account for this phenomenon as well as for the serial correlation and heteroscedasticity. Considering also the previous study of Riveros-Gavilanes (2021), we implemented the test for cointegration on the panel structure for the regions, however, for some of the regions, it was computationally infeasible to provide an estimation given the characteristics of the number of countries N in the sub-samples and the time window T.

Table 7. Summary of Cointegration results by region.

Region	Result	Description
World	Inconclusive	Numerically unfeasible
East Asia and Pacific	Inconclusive	Numerically unfeasible
Europe and Central Asia	Cointegration	Westerlund’s test
Latin America and the Caribbean	Cointegration	Kao’s test
The Middle East and North Africa	Inconclusive	Numerically unfeasible
North America	Cointegration	Kao & Pedroni tests
South Asia	Cointegration	Westerlund’s test
Sub-Saharan Africa	Inconclusive	Numerically unfeasible

Source: Own Elaboration

Given the massive number of countries around the world (more than 140 countries) the cointegration test at the world level did not provide any computational result (as expected), but for single regions, evidence of cointegration was only found in 1) Europe and Central Asia, 2) Latin America and the Caribbean, 3) North America, and 4) South Asia. Hence, for the regions in which cointegration was found, we estimated the error correction models to examine long and short-run dynamics. For the countries that did not exhibit cointegration, we ran only the Sen’s SWF in levels using the robust standard errors for heteroscedasticity, cross-sectional dependence, and serial correlation. Regression results appear in Table 8

Table 8. Regressions for the regions without evidence of cointegration.

	(1)	(2)	(3)
VARIABLES	EAST ASIA AND PACIFIC	MIDDLE EAST AND NORTH AFRICA	SUB-SAHARAN AFRICA
ln_GDP_cap	0.234***	0.284***	0.226***
	(.0066975)	(.0208587)	(.0431859)
ln_Gini_complement	0.019	0.2047*	-0.0574
	(.0253876)	(.0977)	(.0343061)
Constant	-2.647***	-3.839***	-2.103***
	(.133474)	(.3583963)	(.4070807)
Observations	84	56	37
Number of groups	9	9	6
Within R-Squared	0.966	0.8782	77.37

Note: Driscoll-Kraay's robust standard errors were used for the fixed effects regressions. The Dependent variable is ln_HDI. Controls IncludedStandard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1 Source: Own Elaboration

Regressions across the regions present asymmetric effects of the income levels and the complement of Gini's coefficient related to the levels of welfare. East Asia and the Pacific region with Sub-Saharan Africa do not have enough statistical confidence to indicate that the levels of equality (given by the complement of the Gini distribution) are correlated with the welfare (measured by the human development index). The size of the elasticities varies across the regions, and the only Middle East and North Africa have the complement of the Gini's coefficient statistically significant at a 10% in comparison. However, the results suggest that all regions on average exhibit an increasing trend in income levels rather than improvements in the income distribution.

For the regions in which there was evidence of cointegration, we present the results segmented in two ways, the first using the long-run equations and the second, using the error-correction models. The long-run equations for the Sen's SWF present non-significant results for the complement of the Gini coefficient, except for the region of Latin America and the Caribbean, which according to the estimations is the only region that contains statistical significance at a 1% to be related with improvements on the welfare when the income distribution improves (captured by the natural logarithm of the Gini's complement). For the other regions of Europa and central Asia, North America and South Asia, the complement of the Gini coefficient is not statistically significant at a 10% to explain the levels of welfare, and only for Europe and Central Asia, the income (GDP per capita) does have statistically significance in the long-run. A couple of results have to be noted, the size of the elasticities regarding the GDP per capita do not differ significantly between Europe and Central Asia, and Latin America and the Caribbean, but these do greatly change when they are compared with North America and South Asia, for these last two regions the number of observations is small, hence it is inconclusive to state anything.

Table 9. Long-run Regressions over the Welfare for the regions with evidence of cointegration.

	(1)	(2)	(3)	(4)
VARIABLES	EUROPE AND CENTRAL ASIA	LATIN AMERICA AND THE CARIBBEAN	NORTH AMERICA	SOUTH ASIA
ln_GDP_cap	0.204***	0.211***	0.209	0.138
	(.0083445)	(.0172813)	(.0377794)	(.057277)
ln_Gini_complement	0.011	0.132***	0.395	0.125

	(.0251296)	(.0286993)	(.2280711)	(.1690741)
Constant	-2.31***	-2.764***	-3.983	-2.5881***
	(0.110)	(.0866969)	(1.306756)	(.7540569)
Observations	655	332	29	20
Within R-Squared	0.8302	0.8669	0.8486	0.8893
Number of groups	44	23	2	7

Note: Controls Included in the Fixed Effects Regressions, the dependent variable is the ln HDI. Driscoll-Kraay robust estimates were used. Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1 Source: Own Elaboration

For the short-run, the estimated error-correction models report some stable relationships towards the long-run given the statistical significance of the partial adjustments coefficients in the ECTs. However, results for North America and South Asia are inconclusive given the limited number of observations. In the short-run, Europe and Central Asia, Latin America, and the Caribbean reflect that only the economic growth is statistically significant and also positively correlated with the growth in the welfare, but the growth in the complement of the Gini coefficient does not influence the welfare over the short-run. The speed of adjustment also differs across regions, particularly for Europe and Central Asia, the speed of adjustment is about 8.87% while for Latin America and the Caribbean the short-run adjustment speed is about 9.77%.

Table 10. Short-run ECM Regressions over the Welfare Growth for the regions with evidence of cointegration.

	(1)	(2)	(3)	(4)
VARIABLES	EUROPE AND CENTRAL ASIA	LATIN AMERICA AND THE CARIBBEAN	NORTH AMERICA	SOUTH ASIA
D_ln_GDP_cap	0.0629***	0.0825***	-0.0897	-
	(.0041676)	(.0130561)	(0588187)	(-)
D_ln_Gini_comp	-0.00177	-0.0032	0.06839	-
	(.0063292)	(.0097528)	(.0355908)	(-)
ECT 1	-0.0887***			
	(.0104799)			
ECT 2		-0.09766***		
		(.0138605)		
ECT 3			-0.1078	
			(.0748876)	
ECT 4				-
				(-)
Constant	0.00393***	.0048364***	.0043066	-
	(.0002992)	(.0006174)	(.0006845)	(-)
Observations	571	262	29	2
Number of groups	41	16	2	1

Note: Controls Included. Driscoll-Kraay robust estimates were used. The dependent variable is the log difference of the HDI. Log differences were used as growth approximations for the variables. Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1 Source: Own Elaboration

6 Conclusions

The empirical evidence confirms the heterogeneity of the welfare impact of income inequality across regions. Some regions exhibit low-income inequality levels escorting volatile welfare (like Europe and central Asia). Other regions exhibit high levels of income inequality that are uncorrelated with welfare patterns (Sub-Saharan Africa and Latin America and the Caribbean). Generally speaking and by applying on data from more than 140 countries, both the GDP per capita and the complement of the Gini coefficient are statistically significant at 5% to explain positively the levels of welfare measured by the HDI through the linearization of the Sen's SWF with fixed effects components. The income-welfare elasticity is greater than the equality-welfare elasticity at this level.

Long-run cointegration between income, welfare, and inequality is found only in Europe and Central Asia; Latin America and the Caribbean; North America; South Asia. While for the rest we could not have conclusive findings due to lack of observations or only short-run correlations were documented. A statistically significant correlation between the complement of the Gini coefficient and welfare (proxied by the Human development index) over the short run is found in the Middle East and North Africa, and Latin America and The Caribbean.

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8 Appendixes

8.1 Unit-Root Testing

Variable	Test	Name	Statistic	P-value	Conclusion
Ln GDP per capita	Im-Pesaran-	Shin Z-t-tilde-bar	9.8286	1.0000	Unit-root
	Fisher	Inverse chi-squared	267.0003	0.9987	
		Inverse normal	6.9703	1.0000	
		Inverse logit t	7.2335	1.0000	
		Modified inv. chi-squared	-2.7994	0.9974	
Log Difference GDP per Capita	Im-Pesaran-	Shin Z-t-tilde-bar	-22.7216	0.0000	Stationary
	Fisher	Inverse chi-squared	1273.7900	0.0000	
		Inverse normal	-22.5679	0.0000	
		Inverse logit t	-25.7997	0.0000	
		Modified inv. chi-squared	35.8092	0.0000	
Ln HDI	Im-Pesaran-Shin	Shin Z-t-tilde-bar	-0.0990	*	Unit-root
	Fisher	Inverse chi-squared	422.0065	0.0001	
		Inverse normal	-0.5236	0.3003	
		Inverse logit t	-1.0287	0.1520	
		Modified inv. chi-squared	3.9408	0.0000	
Log Difference HDI	Im-Pesaran-Shin	Shin Z-t-tilde-bar	-21.8094	*	Stationary
	Fisher	Inverse chi-squared	946.3649	0.0000	
		Inverse normal	-15.8475	0.0000	
		Inverse logit t	-18.2970	0.0000	
		Modified inv. chi-squared	24.6034	0.0000	
Ln Gini Complement	Im-Pesaran-Shin	Shin Z-t-tilde-bar	-	-	Unit-root
	Fisher	Inverse chi-squared	122.5327	0.7519	
		Inverse normal	0.3270	0.6282	
		Inverse logit t	0.2189	0.5866	
		Modified inv. chi-squared	-0.7005	0.7582	
Log Difference	Im-Pesaran-Shin	Shin Z-t-tilde-bar	-	-	Stationary

Gini Complement	Fisher	Inverse chi-squared	1558.0787	0.0000	
		Inverse normal	-31.8213	0.0000	
		Inverse logit t	-53.0841	0.0000	
		Modified inv. chi-squared	86.9895	0.0000	

Source: Own Elaboration

8.2 Cointegration tests

Region	Test	Name	Statistic	P-value	Results
World	-		-	-	Numerically unfeasible
East asia and pacific	-		-	-	Numerically unfeasible
Europe and central asia	Westerlund	Variance Ratio	-2.2566	0.0120	Cointegration
Latin america and the caribbean	Kao	Modified Dickey-Fuller	2.4142	0.0079	Cointegration
		Dickey-Fuller	2.0757	0.0190	
		Augmented Dickey-Fuller t	1.8264	0.0339	
		Unadjusted modified Dickey-Fuller	2.2961	0.0108	
		Unadjusted Dickey-Fuller	1.9283	0.0269	
Middle east and north africa	-	-	-	-	Numerically unfeasible
North America	Kao	Modified Dickey-Fuller	-0.9923	0.1605	Evidence of some Cointegration

		Dickey-Fuller	0.1605	0.1427	
		Augmented Dickey-Fuller t	-0.4066	0.3421	
		Unadjusted modified Dickey-Fuller	-1.8239	0.0341	
		Unadjusted Dickey-Fuller	-1.4108	0.0792	
	Pedroni	Modified Phillips-Perron	-0.9381	0.1741	Cointegration
		Phillips-Perron	-3.8519	0.0001	
		Augmented Dickey-Fuller	-2.6756	0.0037	
	Westerlund	Variance ratio	-1.4951	0.0674	Cointegration at 10%
South Asia	Westerlund	Variance ratio	-1.7643	0.0388	Cointegration
Sub-saharan africa	-	-	-	-	Numerically unfeasible

Note: For the numerically unfeasible results, the three types of cointegration were tested (Kao, Pedroni & Westerlund) with different specifications, but by the characteristics of the number of individuals N versus the small number of time periods T this result is expected.
Source: Own Elaboration.